1.4

Water Under the Bridge

Modeling with Functions

LEARNING GOALS

In this lesson, you will:

- Use multiple representations of functions to model and solve problems.
- Use multiple representations of functions to analyze problems.

"It's just water under the bridge" is more than a saying for some hydrologists. To them, it's their career. Some hydrologists specialize in the design of city drainage systems.

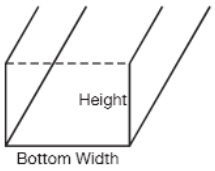
So, you might be asking: how important is a city's drainage system? The key function to any drainage system is to channel rain water out of the area at the maximum speed possible. In the early part of the 20th century, the Los Angeles River routinely jumped its banks causing some areas of the city to flood. Outraged citizens demanded a better means of draining water after torrential rains. Hydrologists at the time decided to convert the Los Angeles River from a natural river to a massive storm drain. By pouring concrete and building up the sides of the drain, the city no longer flooded. However, the water that rushes through the drain can reach speeds of 45 miles per hour. These speeds are obviously very dangerous for anyone who might be in the storm drain system at the time of the storm. So while the drain has helped save the city from destruction caused by flooding, many lives have been lost as a result of citizens and rescuers being swept away in the drain system during a storm.

Do you think the city should raise the height of the drain so fewer people fall in? Would that affect how quickly the water flows through the drain?

PROBLEM 1 The Water's Five Feet High and Rising

A nearby town hired a civil engineer to rebuild their storm drainage system. The drains in this town are open at the top to allow water to flow directly into them. While designing the drains, the engineer must keep in mind the height and the width of the drain. She needs to consider the height because the water cannot rise above the drain or it will flood the town and cause major destruction. However the drain must also be wide enough that it will not get clogged by debris.

The civil engineer will use rectangular sheets of metal to build the drains. These sheets are bent up on both sides to represent the height of the drain. An end view of the drain is shown.



1. Use a sheet of paper to model a drain.
   a. Compare your model of a drain to your classmate's models. Identify similarities and differences between your models.

   b. How does folding the sides of the drain affect the bottom width of the drain?

   c. Describe the drain that you think best fits the needs of the town. Explain your reasoning.

The sheets of metal being used to create the drain are 8.5 feet wide. The engineer wants to identify possible heights and bottom width measurements she could use to construct the drains.

1

2. Determine the bottom width for each given height. Then complete the table by choosing different heights and calculating the bottom widths for those heights. If necessary, construct models of each drain.

Height of the Drain (feet)	Bottom Width of the Drain (feet)
0	
1.5	
3	

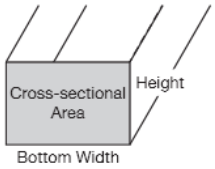
3. Describe how to calculate the bottom width for any height.

4. Define a function $w(h)$ for the bottom width given a height of h feet.

5. The engineer needs to identify the measurements that allow the most water to flow through the drain. What does the engineer need to calculate? What does she need to consider?

1

In order to determine the drain dimensions that allows the most water to flow through, the engineer must calculate the cross-sectional area. The cross-sectional area of a drain is shown.



6. Describe how to determine the cross-sectional area of any drain.

7. Predict and describe the drain with the maximum cross-sectional area.

8. Define a function $A(h)$ for the cross-sectional area of the drain with a height of h feet.

9. Use a graphing calculator to graph the function $A(h)$. Label your axes.

10. Analyze your graph.

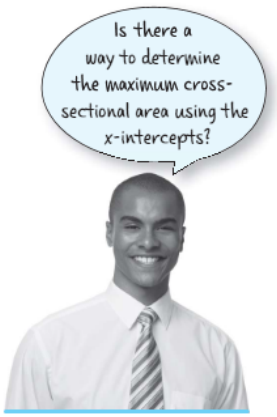
a. What is the maximum cross-sectional area for the drain pipe? Explain your reasoning.

1

b. Identify the intercepts of $A(h)$. What does each mean in terms of this problem situation? Label each intercept on the graph.

c. Identify the equation of the axis of symmetry. Then label the axis of symmetry on the graph. Finally, describe the relationship between the axis of symmetry and the maximum cross-sectional area.

11. Draw and label the drain with the greatest cross-sectional area.



1

12. In this problem you built a new function $A(h) = h(8.5 - 2h)$ using two existing functions.

a. What is the first factor in this function? What does it represent in terms of this problem situation?

b. What is the second factor in this function? What does it represent in terms of this problem situation?

c. Identify the function families represented by each factor.

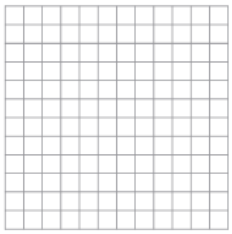
d. When these factors are multiplied together what type of function is created? Why does this happen?

PROBLEM 2 Determine the Best Design

1

A civil engineering company is hired to design a new drainage system for your town. To construct one of the storm drains, a sheet of metal that is 15.25 feet wide is folded on both sides.

Describe the drain that has the maximum cross-sectional area. Include at least two different representations in your description. Show all work and explain your reasoning.



Be prepared to share your solutions and methods.